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BREAKUP CRITERIA FOR FLUID PARTICLES

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Abstract—A simple mechanistic model is developed based on the combination of Kelvin–Helmholtz and Rayleigh–Taylor instability theory to describe the breakup of freely rising or falling fluid particles, i.e. bubbles and drops. The breakup is predicted to occur if the growth rate of interfacial waves on the leading front is faster than the rate at which waves propagate around the interface to the side of the particle. Based on this theoretical model and available experimental data, simple correlations are developed to predict the maximum size a fluid particle can reach. Predicted values of the breakup diameter are compared with experimental data for cases of freely rising bubbles, falling drops in gas and freely falling or rising drops in immiscible liquids. The results are extended to predict the maximum droplet size in a high-velocity gas field which is the most interesting case in terms of practical applications. Good agreement between predicted values and experimental data indicates that the principal mechanisms involved in the fluid particle breakup process are properly accounted for by the proposed model.

Key Words: bubbles and drops, fluid particle breakup size, maximum diameter of bubbles and drops

INTRODUCTION

Breakup size of fluid particles, bubbles and drops, in dispersed two-phase flow systems including the liquid–liquid particulate systems is an important factor in determining the fluid particle size distribution and, hence, the effectiveness of the interfacial transport of mass, momentum and energy. A knowledge of disintegration of bubbles and drops is essential to the eventual understanding of the interfacial transfer mechanisms and two-phase flow-pattern transitions in a large number of engineering applications. These include: gas–liquid droplet systems, such as atomizers, dryers, absorbers, wet steam separators and cryogenic heat exchangers: liquid–liquid droplet systems, such as liquid–liquid extractors, separators used with distillation columns and packed towers when the packing is not wetted by the dispersed phase; and finally, liquid–gas (or vapor–bubbly) systems, such as boiling water and pressurized boiling water reactors, boilers, flash distillation and aeration units, etc. Although drops and bubbles seldom occur in isolation in such systems, it is essential to understand the behavior of a single fluid particle before a full knowledge of interacting bubbles and drops can be achieved.

For the purpose of providing basic information on the maximum size a fluid particle can reach, a number of processes which may cause breakup of fluid particles have been identified in the literature. The most important breakup mechanisms are classified as rapid accelerations (Hinze 1955), high shear stresses (Taylor 1934) and turbulent fluctuations (Sleicher 1962).

In the foregoing breakup mechanisms, disturbances which cause fluid particle splitting are due to rapid acceleration, high shear stresses and turbulent fluctuations in the continuous surrounding fluids. It has been observed that even when no such external disturbance is present, there is a limit to the size to which bubbles and drops can reach. The maximum size attained by a single bubble or drop rising or falling freely through stagnant media in the absence of such disturbances has been attributed to Rayleigh–Taylor instability. This type of breakup mechanism was first considered by Komaboyashi *et al.* (1964) to determine the maximum size of falling drops in air. This theory has been extended over the years by Blanchard (1962), Cotton & Gokhale (1967). Klett (1971), Hendricksen & Ostergaard (1974) and Grace and co-workers (Clift & Grace 1973; Clift *et al.* 1974, 1978). For example Grace *et al.* (1978) developed a semi-empirical relation to predict the maximum particle diameter in which a constant was correlated using existing experimental data. It was found that the data for bubbles requires a different constant, 3.8, than the data for liquid drops. For the latter case, the optimum value of the constant was found to be 1.4.

It is important to note that in this type of analysis the breakup criteria were based on the growth of the standing waves, i.e. Rayleigh–Taylor instability, where there is no relative velocity permitted between the particulate and continuous phases. However, in reality, even for the breakup in the stagnant media there exists a relative motion between particulate and continuous fluids, and the growth of disturbances generated at the interface depends on the magnitude of the relative velocity. Therefore, the use of the Rayleigh–Taylor instability analyses seems inconsistent in this case. This is particularly true for falling drops in a gaseous media where the relative velocity can be very high. It is natural to expect an effect of the relative velocity on the wave propagation and breakup processes.

Kelvin–Helmholtz theory allows a relative motion between two superposed fluid layers. Disturbances generated by this instability propagate at the interface with a certain speed while they grow or decay. Furthermore, since the breakup of fluid particles proceeds from the advancing interfacial surface, i.e. from the upper surface for rising bubbles and drops from the lower surface for falling drops, it is natural to expect both the Kelvin–Helmholtz and Rayleigh–Taylor instabilities to become effective in the breakup process.

In veiw of the above discussion, this study has three objectives. Based on the combination of Kelvin–Helmholtz and Rayleigh–Taylor instability theories, the first objective is to develop a unique method to describe the breakup of fluid particles. The method thus developed is unique in the sense that it can be used for predicting the breakup diameter of rising bubbles as well as falling or rising drops in a gas or in an immiscible different liquid. The second objective is to develop a series of simple correlations to determine the maximum size a fluid particle can reach. Finally, the third objective is to extend the theory to predict the maximum droplet size in a high-velocity gas stream, which is the most interesting case in terms of practical applications. This extension is possible since the mechanistic model developed here partly depends on the Kelvin–Helmholtz instability theory.

BREAKUP ANALYSIS

Modeling

The stability of the two superposed incompressible, inviscid fluids to be considered here is illustrated in figure 1. The lower fluid is identified by subscript 1 and the upper fluid by 2. The fluids are flowing concurrently in a horizontal, constant area channel. The velocities of the two fluids are assumed to be horizontal in direction, and denoted by u_1 and u_2, respectively. Assuming the perturbed flow is irrotational, and following the derivations of Chandrasekhar (1968), Lamb (1945) and Yih (1980), the speed of propagation c_r and the growth factor kc_i can be expressed as follows:

$$c_r = \frac{\rho_1 \coth(kh_1)\, u_1 + \rho_2 \coth(kh_2)\, u_2}{\rho_1 \coth(kh_1) + \rho_2 \coth(kh_2)} \qquad [1]$$

and

$$kc_i = \left\{ \frac{\rho_1 \rho_2 \coth(kh_1) \coth(kh_2)\, (u_1 - u_2)^2 k^2}{[\rho_1 \coth(kh_1) + \rho_2 \coth(kh_2)]^2} - \frac{\sigma k^3 - g\,(\rho_1 - \rho_2) k}{[\rho_1 \coth(kh_1) + \rho_2 \coth(kh_2)]} \right\}^{1/2}. \qquad [2]$$

where, as illustrated in figure 1, $h_1$and h_2 represent the fluid thickness of the lower and upper fluids, respectively, and k is the wavenumber, $k = 2\pi/\lambda$. In deriving this equation, it was assumed that

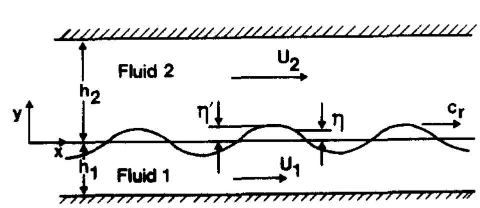

Figure 1. Stability of two superimposed fluids flowing concurrently in a constant cross-sectional area channel.

the interfacial perturbation is periodic in x. Thus,

$$\eta = \eta_i \exp[ik(x - ct)], \tag{3}$$

where η_i is the perturbation amplitude at the interface, and c is the complex wave celerity defined as

$$c = c_r + ic_i. \tag{4}$$

The growth factor, kc_i, determines the degree of amplification or damping. The interfacial disturbances are damped if $kc_i < 0$ and the mean flow configuration is stable, the disturbances are amplified if $kc_i > 0$, and the mean flow is unstable. Finally, the mean flow is said to be neutrally stable if $kc_i = 0$.

For the purpose of using the plane flow Kelvin–Helmholtz theory summarized above to describe the breakup of fluid particles, we now consider a cap bubble rising in a liquid, as illustrated in figure 2. Here a cap bubble is chosen for the purpose of reference. The present theory will be equally applicable to rising or falling drops with spherical or ellipsoidal shapes. Identifying the continuous and dispersed fluids by subscripts c and d, respectively, the plane flow stability results summarized by [1] and [2] can be adopted for the cap bubble case as follows:

$$c_r = \frac{3}{2}\frac{\rho_c u_c \sin\theta}{\rho_c + \rho_d \coth(kh_d)} \tag{5}$$

and

$$kc_i = \left\{\frac{\rho_c \rho_d \coth(kh_d) k^2 (1.5 u_c \sin\theta)^2}{[\rho_c + \rho_d \coth(kh_d)]} - \frac{\sigma k^3 - g|\Delta\rho| k}{\rho_c + \rho_d \coth(kh_d)}\right\}^{1/2}. \tag{6}$$

In arriving at [5] and [6] from [1] and [2], respectively, the following approximations are introduced:

(a) The effects of viscosity in both dispersed and continuous fluids are neglected. Hence, the breakup criteria will not be expected to hold for extremely viscous fluids.

(b) The breakup of fluid particles proceeds from the advancing interfacial surface, i.e. from the upper surface for rising bubbles and drops and from the lower surface for falling drops, which is in agreement with most experimental observations (Grace *et al.*, 1978). Hence it is assumed here that it will always be the advancing interface of a moving particle that is prone to instability. In view of this assumption, u_1 and u_2 in [1] and [2] are interpreted as the tangential velocity components, $u_{d\theta}$ and $u_{c\theta}$, respectively.

(c) The effect of advancing front curvature is neglected except insofar as it determines the value of the tangential velocity component. It can be argued that

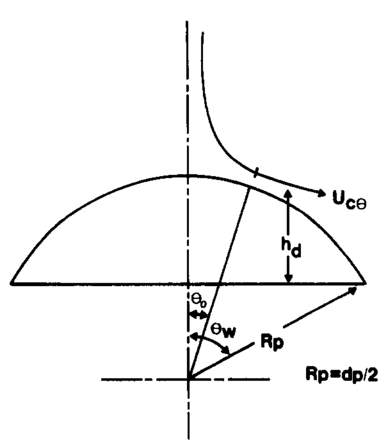

Figure 2. Schematic illustration of flow around a rising cap bubble.

such an effect is of minor consequence for drops and bubbles which are sufficiently large for breakup to be a factor.

(d) The circulation within the fluid particle is neglected. This implies the presence of enough surface-active contaminants and a sufficient surface-active contaminant concentration gradient to damp out the internal circulation. Systems which exhibit high surface tension, including common fluid pairs like air–water, liquid metals–air and aqueous liquid–nonpolar liquids, are subject to these effects. In view of these assumptions and using the potential flow theory for flow around a spherical particle, $u_{c\theta}$ and $u_{d\theta}$ are approximated by

$$u_2 \rightarrow u_{c\theta} = \tfrac{3}{2} u_c \sin\theta; \qquad u_1 \rightarrow u_{d\theta} \simeq 0. \quad [7]$$

(e) It is noted that the surrounding fluid dimension is much larger than the particle size. Thus, $h_2 \rightarrow h_c \rightarrow \infty$, and for large amounts $\coth(kh_c)$ is approximated by

$$\coth(kh_c) \simeq 1.0. \quad [8]$$

It is evident from [5] and [6] that the speed of propagation as well as the growth factor depend upon the local angular position, original disturbance location and the dispersed phase fluid thickness at the origination of the disturbances. Referring to figure 2, it can be shown that h_d is given by the following equation:

$$h_d = \frac{d_p}{2}(\cos\theta_o - \cos\theta_\omega). \quad [9]$$

Here d_p is given in terms of the mean radius of curvature, $d_p = 2R_p$; and θ_o describes the angular position at which the disturbances originate.

Equation [6] represents the growth factor of the combined Kelvin–Helmholtz and Rayleigh–Taylor instabilities as applied to a rising cap bubble. It should be emphasized here that the above stability criterion represents only the first step in developing a correlation for the breakup of a fluid–particle interface. This information simply indicates when these interfacial waves occur and what their growth rates are. However, the appearance of the wave on the interface does not necessarily imply that it leads to drastic changes at the interface such as the breakup of particles. To answer this question of whether the waves can lead to a breakup or not, it is necessary to know the time required for these waves to grow to a certain amplitude so that splitting eventually can occur.

Breakup mechanism

A mathematical model is proposed here to predict the point at which breakup will be attained under given conditions. If t_g denotes the growth time at which the instability at the interface lead to a breakup, t_g can be calculated from the assumed wave form given by [3]. Thus,

$$t_g \sim \frac{1}{kc_i}. \quad [10]$$

For the purpose of convenience a proportionality factor, C_g, may be introduced as follows:

$$t_g = \frac{C_g}{kc_i} \qquad \text{and} \qquad t_g = C_g t'_g, \quad [11]$$

where $t'_g \equiv 1/kc_i$. In view of [6], t'_g can be expressed as

$$t'_g = \left\{\frac{\rho_c\rho_d\coth(kh_d)\,(1.5\,u_c k\sin\theta_o)^2}{[\rho_c + \rho_d\coth(kh_d)]^2} - \frac{\sigma k^3 - g\,|\Delta\rho|\,k}{\rho_c + \rho_d\coth(kh_d)}\right\}^{-1/2}. \quad [12]$$

Disturbances originate near the top of the roof of a bubble and propagate down to the periphery with the local speed of propagation, c_r. In practice a bubble does not split unless the disturbance had grown sufficiently before the tip of the growing spike reaches the side of the bubble. If the wave travels to the end of a cap bubble or to the equator of a spherical particle without causing a breakup, it will be swept away at the edge into the continuous fluid. An estimate of the likelihood of splitting may be obtained by comparing the time required for a disturbance to grow with the time available for the growth. If t_p represents the propagation time, that is the time required for a disturbance to travel from its origination to the side of the bubble, t_p can be calculated by

$$t_p = \int_{\theta_o}^{\theta_\omega} \left(\frac{d_p}{2c_r}\right) d\theta = \left[\frac{\rho_c + \rho_d \coth(kh_d)}{3\rho_c u_c}\right] d_p \ln\left[\frac{\left|\tan\left(\frac{\theta_\omega}{2}\right)\right|}{\left|\tan\left(\frac{\theta_o}{2}\right)\right|}\right]. \qquad [13]$$

where θ_o is the angular position where the disturbances initiate.

The likelihood of a breakup may now be assessed by comparing the values of t_g and t_p. The model proposed here postulates that fluid particles are likely to split if the time available for disturbances to grow (i.e. the propagation time, t_p) is sufficiently large relative to the growth time, t_g. Accordingly, a fluid particle tends to breakup due to a disturbance if

$$t_p \geqslant t_g \quad \text{or} \quad \frac{t_p}{t'_g} \geqslant C_g \quad \text{with} \quad t_g \equiv C_g t'_g. \qquad [14]$$

Combining [12] and [13] with [14], the breakup criterion may be expressed as

$$\left[\frac{\rho_c + \rho_d \coth(kh_d)}{3\rho_c u_c}\right] d_p \ln\left[\frac{\left|\tan\left(\frac{\theta_\omega}{2}\right)\right|}{\left|\tan\left(\frac{\theta_o}{2}\right)\right|}\right] \times \left\{\frac{\rho_c \rho_d \coth(kh_d)\,(1.5\,u_c k \sin\theta_o)^2}{[\rho_c + \rho_d \coth(kh_d)]^2} - \frac{\sigma k^3 - g\,|\Delta\rho|\,k}{\rho_c + \rho_d \coth(kh_d)}\right\}^{1/2} \geqslant C_g. \qquad [15]$$

The breakup criterion developed here is similar to that used previously by Grace *et al.* (1978). However, the approach moves away from previous analyses, which were largely based on Rayleigh–Taylor instability. Here disturbances generated at the interface grow much more rapidly due to the relative velocity, and the degree of instability affected by the relative motion between two phases.

Assuming that the wake angle, θ_ω, the angular position of the initial disturbance, θ_o, the terminal velocity u_c and the amplitude ratio, C_g, are expressible in terms of the particle diameter and the wavenumber, basically there will be only two variables in [15], namely the wavenumber and the particle diameter. Then, once k is specified, d_p can be calculated from [15]. The evaluation of these parameters is discussed below.

Wake angle, θ_ω

Large fluid particles which are prone to splitting have been studied in some detail previously, and several transition criteria for fluid particle shape regimes have been proposed by Clift *et al.* (1978). When these studies are compared with available experimental data it is seen that drops falling in gases and drops falling or rising in another liquid never reach the spherical-cap particle regime. However, very large bubbles, in the order of centimeters, and most bubbles at the breakup point attain the spherical-cap shape.

Attempts to predict wake angles theoretically for spherical-caps have met with only limited success (Moore 1959; Rippin & Davidson 1967). Based on experimental observations Clift *et al.* (1978) recommended the following empirical equation for bubbles:

$$\theta_\omega = 50 + 190 \exp(-0.62\,\mathrm{Re}_c^{0.4}), \qquad [16]$$

where θ_ω is expressed in degrees and $\mathrm{Re}_c \equiv \rho_c u_c d_e/\mu_c$ is the continuous phase Reynolds number. In the present analysis [16] is used for bubbles, and $\theta_\omega \simeq 90°$ is used for liquid droplets falling or rising in another fluid.

Angular position of initial disturbance generation, θ_o

From [13] it is evident that disturbances which originated at the axis of symmetry, i.e. at $\theta_o = 0$, would never reach the side of the cap bubble or the equator of spherical particles. They are purely standing waves in nature. Observations of splitting bubble experiments performed by Clift *et al.* (1974) indicated that disturbances usually develop in a regular pattern on either side of the leading nose. Clift *et al.* suggested that the bubble is a node when the initial disturbance originates, then,

$$\theta_o = \frac{\lambda}{2d_p} = \frac{\pi}{kd_p}. \qquad [17]$$

Consistent with Clift *et al.*'s suggestion, [17] will be used throughout the analysis.

Terminal velocity, u_c

There is a substantial body of data in the literature on the terminal velocity of a single bubble or drop. From these data many correlations for calculating the velocity, u_c, have been developed (Hu & Kintner 1985; Klee & Treyball 1956; Mendelson 1967; Marruici *et al.* 1970; Wallis 1974; Grace *et al.* 1976). Similar studies have also been carried out for multiparticle systems by Ishii & Zuber (1979). The terminal velocity correlations were reviewed in detail by Grace *et al.* (1976). Here, the correlations recommended by Grace *et al.* are used. These are summarized below.

(1) For large bubbles rising through a liquid,

$$u_c = \left(\frac{2}{3}\right)\left(\frac{g\,|\Delta\rho|\,d_p}{2\rho_c}\right)^{1/2}. \qquad [18]$$

(2) For drops falling through a gas,

$$u_c = 2.0\left(\frac{g\,|\Delta\rho|\,\sigma}{\rho_c^2}\right)^{1/4}. \qquad [19]$$

(3) For drops rising or falling through a liquid,

$$u_c = 0.5\left(\frac{\mu_c}{\rho_c d_e}\right)[(F^2 + 2\mathrm{Ar})^{1/2} - F], \qquad [20]$$

where Ar is the Archimedes number. It is defined as

$$\mathrm{Ar} \equiv \frac{g\,|\Delta\rho|\,\rho_c d_e^3}{\mu_c^2} \qquad [21]$$

and the parameter F is defined as

$$F \equiv \frac{3\left[2 + 3\left(\dfrac{\mu_d}{\mu_c}\right)\right]}{\left(1 + \dfrac{\mu_d}{\mu_c}\right)}.$$

In the above expressions d_e is the volume equivalent diameter. As in the case of the empirical correlations documented above, in most drop or bubble experiments, data are tabulated in terms of the volume equivalent diameter, d_e, rather than the mean curvature diameter or the particle diameter, d_p. Therefore, it is desirable to express the equations in terms of d_e. A relation may be given in the form

$$d_p = C_e d_e \quad \text{with} \quad C_e = \left[\frac{4}{(1-\cos\theta_\omega)^2(2+\cos\theta_\omega)}\right]^{1/3}. \qquad [22]$$

Once θ_ω, θ_o and u_c are determined in terms of k and d_p (or d_e) it is evident from [15] that in order to arrive at a predictive criterion for d_p, one still needs to know k and C_g.

Wavenumber, k

The interface between the dispersed and continuous phases is unstable only for $kc_i \geqslant 0$. In view of [6] the neutral stability condition can be expressed as

$$\frac{\rho_d \coth(kh_d)\,(1.5\,u_c \sin\theta_o)^2 k}{\rho_c + \rho_d \coth(kh_d)} - \frac{\sigma k^2 - g\,|\Delta\rho|}{\rho_c} \geqslant 0. \qquad [23]$$

The wavenumber determined from this condition sets an upper limit on wavenumbers which need to be considered. The leading surface of a bubble or drop may therefore become unstable if the wavenumber of a disturbance at the interface is less than a critical value, $k < k_{cr}$, where k_{cr} is determined from [23]. Thus, $k_{max} = k_{cr}$.

There is also an upper limit on the wavelength, λ, or a lower limit on the wavenumber, imposed by the fact that a disturbance, if λ were too large, represents a gross deformation of the bubble or drop and not a perturbation of the leading interface (Grace *et al.* 1978). A reasonable upper limit corresponds to half the circumference of the fluid particle. Hence, for a cap bubble of wake angle θ_ω, the maximum wavelength becomes

$$\lambda_{\max} = \theta_\omega d_p \quad \text{or} \quad k_{\min} \equiv \frac{2\pi}{\lambda_{\max}} = \frac{2\pi}{\theta_\omega d_p}. \qquad [24]$$

Equations [23] and [24], respectively, put a higher and a lower limit on the acceptable values of wavenumbers. Hence instability occurs for some k, such that

$$\frac{2\pi}{\theta_\omega d_p} \leqslant k \leqslant k_{cr}. \qquad [25]$$

Breakup correlation

Variations of t'_g and t_p, as calculated from [12] and [13], respectively, are illustrated in figures 3–5 for some of the fluid pairs. It is to be noted that the experimental values of breakup diameter are used to construct these figures. These three sample figures represent the basic characteristics of other data available in open literature. For example, figure 3 represents the basic characteristics of bubbles, whereas the rest represent drops in gases and liquids. For convenience, the (t_p/t'_g) ratio is also given in these figures.

It is interesting to note that t_p is always greater than t'_g in the acceptable range of wavenumbers, as given by [25], except in a very narrow range close to $k_{\max} = k_{cr}$, where the growth factor, $k_{ci} \to 0$, and, hence $t'_g \to \infty$, which makes the ratio (t_p/t'_g) approach zero. It is noted, however, that this range corresponds to very small wavelengths for which the linearized stability analysis is not expected to hold. Furthermore, it is to be noted that the minimum value of (t_p/t'_g) which is required by [14] can only be achieved at the minimum value of the wavenumber. Although it is usual practice in linearized stability analysis to consider the wavenumber which causes the most unstable wave growth, determined by the root of $d(kc_i/dk) = 0$, the most unstable wave for liquid drops (as

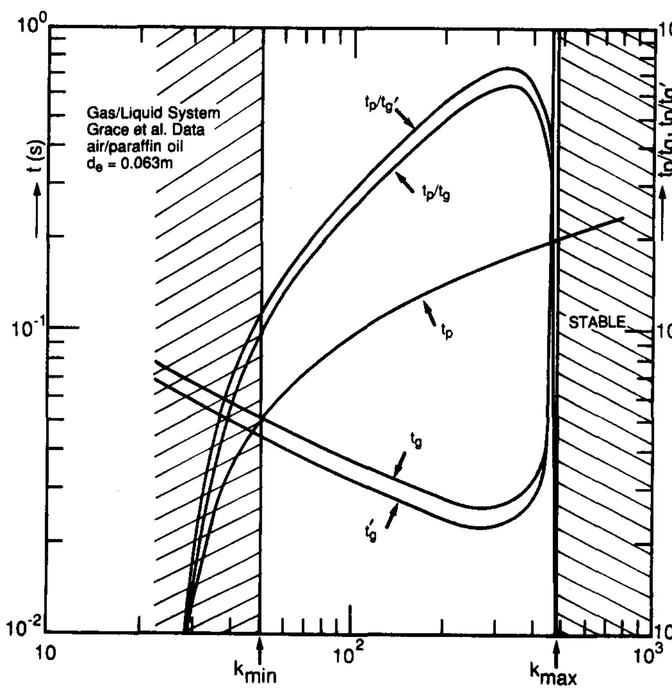

Figure 3. Variation of growth time, t_g, propagation times, t_p, and time ratio, t_p/t_g, for a bubble at $d_e = 0.063$ m as a function of wavenumber, k.

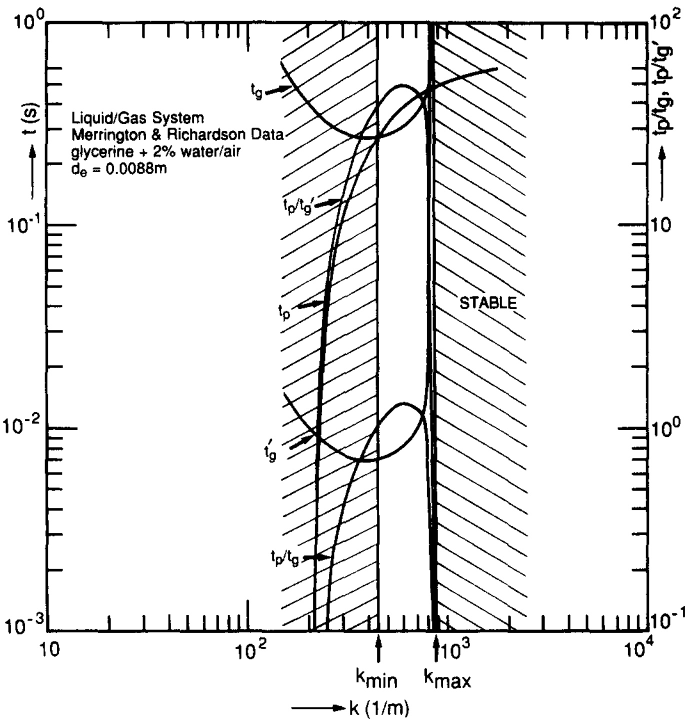

Figure 4. Variation of growth time, t_g, propagation time, t_p, and time ratio, t_p/t_g, for a drop in air at $d_e = 0.0088$ m as a function of wavenumber, k.

illustrated by figures 4 and 5) falls into the unacceptable wavenumber range expressed by [25]. In other words the most unstable wavenumber is so small that the corresponding wavelength, $\lambda = 2\pi/k$, becomes longer than one-half of the circumference. This implies a gross motion of the bubble or drop and not a perturbation of the leading interface. Such a disturbance is considered not to cause particle disintegration. Therefore, instead of the most unstable wave, we propose here to consider the wave which yields the minimum value of (t_p/t_g'), as required by [14].

Using the value $k = k_{min}$ obtained from [24] and the θ_o value calculated from [17], the breakup criterion may be expressed from [15] in dimensionless form as follows:

$$(1+\rho^*)^{1/2} \ln \left[\frac{\left|\tan\left(\frac{\theta_\omega}{2}\right)\right|}{\left|\tan\left(\frac{\theta_\omega}{4}\right)\right|}\right] \left\{2.25\left(\frac{2\pi}{\theta_\omega}\right)^2\left(\frac{\rho^*}{1+\rho^*}\right)\sin^2\left(\frac{\theta_\omega}{2}\right) + \frac{1}{\mathrm{We}}\left[\left(\frac{2\pi}{\theta_\omega}\right)C_e d_e^{*2} - \frac{1}{C_e}\left(\frac{2\pi}{\theta_\omega}\right)\right]^3\right\}^{1/2} \geqslant 3\,C_g, \qquad [26]$$

where d_p^*, We and ρ^* are the dimensionless particle diameter, Weber number and density ratio, respectively. They are defined as

$$d_e^* \equiv \left(\frac{g\,|\Delta\rho|\,d_e^2}{\sigma}\right)^{1/2}, \quad \mathrm{We} \equiv \frac{\rho_c d_e u_c^2}{\sigma}, \quad \rho^* \equiv \frac{\rho_d \coth(k_{min} h_d)}{\rho_c}. \qquad [27]$$

where d_p^*. We and ρ^* are the dimensionless particle diameter, Weber number and density ratio, respectively. They are defined as

$$d_e^* \equiv \left(\frac{g\,|\Delta\rho|\,d_e^2}{\epsilon}\right)^{1/2}, \quad \mathrm{We} \equiv \frac{\rho_c d_e u_c^2}{\sigma}, \rho^* \equiv \frac{\rho_d \coth(k_{min} h_d)}{\rho_c}. \qquad [27]$$

The term $(k_{min} h_d)$ appearing in [27] can be expressed from [9] and [17] in terms of θ_ω. Thus,

$$k_{min} h_d = \left(\frac{2\pi}{\theta_\omega}\right) \sin\left(\frac{3\theta_\omega}{4}\right) \sin\left(\frac{\theta_\omega}{4}\right). \qquad [28]$$

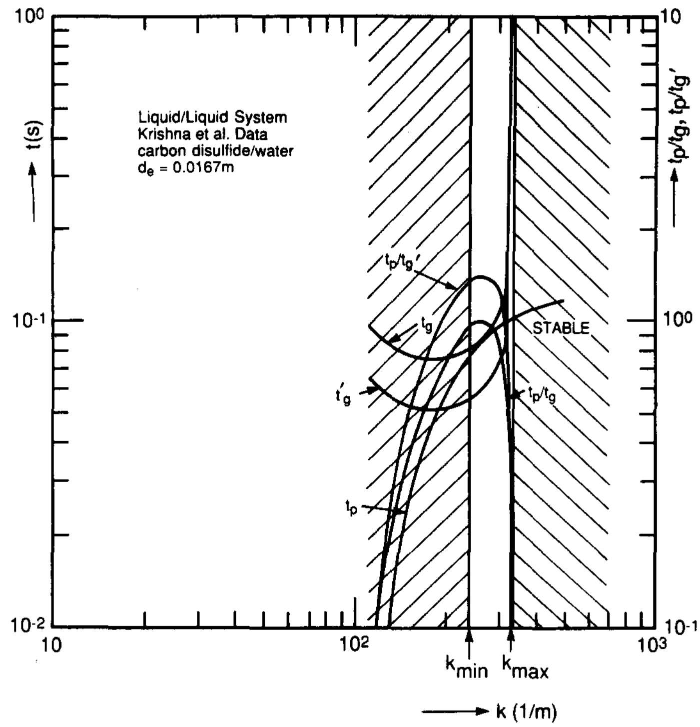

Figure 5. Variation of growth time, t_g, propagation time, t_p, and time ratio, t_p/t_g, for a drop in a liquid at $d_e = 0.0167$ m as a function of wavenumber, k.

It is evident from [26] that in order to arrive at a predictive criterion, one needs to know C_g. In a linearized stability analysis, as is the case here, there exists no analytical way to predict the value of C_g on a purely theoretical basis. It is necessary to resort to experiments. A reasonable approach is to correlate this term in terms of basic variables affecting the breakup process.

Observations on a large number of figures, such as figures 3–5, regarding the numerical value of (t_p/t_g') at $k = k_{min}$ lead to the following conclusions:

(1) For bubbles, (t_p/t_g') is in the order of unity.

(2) For drops, $(t_p/t_g') \sim (1+\rho^*)^m$ with m being in the order of 0.5 at $k = k_{min}$.

Since for bubbles $1+\rho^* \simeq 1$, these observations lead to the conclusion that at $k = k_{min}$, $C_g \simeq (1+\rho^*)^{1/2}$.

Considering the magnitude of the variations in the dimensionless density ratio group from bubbles to drops falling in gases, the emergence of such a dimensionless group as a correlation parameter is not surprising. Furthermore, noting that viscous effects were completely neglected in the present analysis, it is reasonable to use another correlation parameter representative of the viscous effects. A correlation in the following form is sought:

$$\frac{C_g}{(1+\rho^*)^{1/2}} = f(\rho^*, \mathrm{N}\mu_c), \tag{29}$$

where $\mathrm{N}\mu_c$ is the viscosity number of the continuous phase. It is defined as

$$\mathrm{N}\mu_c \equiv \left[\frac{\mu_c^2}{\rho_c\left(\dfrac{\sigma^3}{g\Delta\rho}\right)^{1/2}}\right]^{1/2}. \tag{30}$$

Using the substantial amount of data covering a wide range of fluid properties the functional dependence of C_g on ρ^* and $\mathrm{N}\mu_c$ is determined by linear regression analysis. It is given by

$$\frac{C_g}{(1+\rho^*)^{1/2}} = 0.348\left(\frac{2+3\rho^*}{1+\rho^*}\right)^{1.135}(1+\mathrm{N}\mu_c)^{0.18}. \tag{31}$$

In view of [31] the growth time predicted from [14] is also illustrated in figures 3–5. It may be

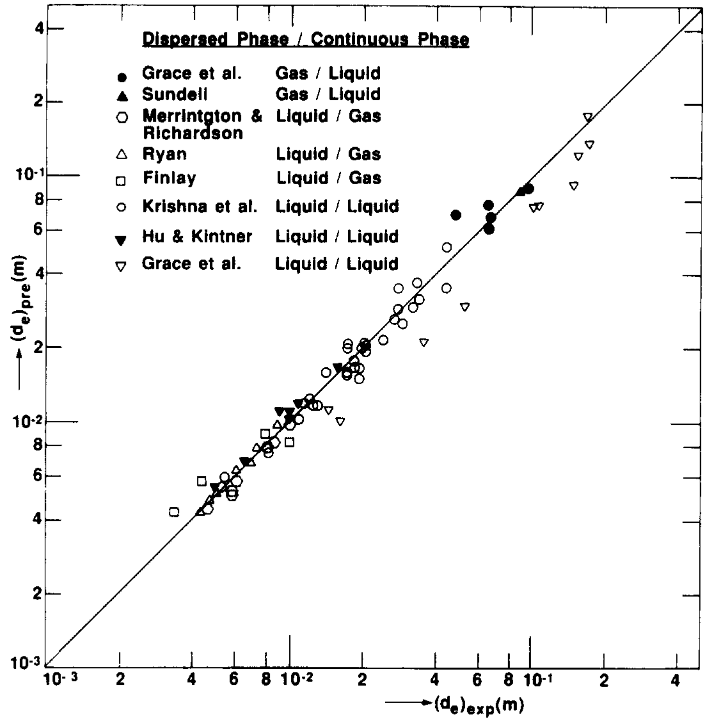

Figure 6. Comparison of predicted breakup diameters with experimental data.

observed from these figures that at $k = k_{\min}$, $t_p/t_g \simeq 1.0$. Furthermore, using [31] in [26] the general breakup criterion becomes

$$2.25\left(\frac{2\pi}{\theta_\omega}\right)^2\left(\frac{\rho^*}{1+\rho^*}\right)\sin^2\left(\frac{\theta_\omega}{2}\right)+\frac{1}{\mathrm{We}}\left[\left(\frac{2\pi}{\theta_\omega}\right)C_e\,d_e^{*2}-\left(\frac{1}{C_e}\right)\left(\frac{2\pi}{\theta_\omega}\right)^3\right]$$
$$\geqslant 1.09\left(\frac{2+3\rho}{1+\rho^*}\right)^{2.27}(1+\mathrm{N}\mu_c)^{0.36}\left\{\ln\left[\frac{\left|\tan\left(\frac{\theta_\omega}{2}\right)\right|}{\left|\tan\left(\frac{\theta_\omega}{4}\right)\right|}\right]\right\}^{-2}. \qquad [32]$$

Equation [32] can be used to predict the maximum fluid particle diameter at breakup. The breakup diameter suggested here is general in the sense that it is applicable for gas–liquid bubbly systems as well as liquid–liquid and liquid–gas droplet systems for relatively low viscosity fluids. It is important to note that the basic parameters affecting the breakup are $\mathrm{N}\mu_c$, We, ρ^* and θ_ω. Since θ_ω depends on Re_c by [19] it can be replaced by Re_c. The effects of these groups on d_e^* will be explored in detail in the subsequent sections.

COMPARISON BETWEEN THEORETICAL PREDICTIONS AND EXPERIMENTAL BREAKUP DATA

Predicted values $(d_e)_{\max}$ obtained from [32] are compared with experimental values in figure 6. The results include: the data of Hu & Kintner (1955), Krishna *et al.* (1959) and Grace *et al.* (1978) for liquid–liquid systems; the data of Merrington & Richardson (1947), Finlay (1957) and Ryan (1978) for liquid drops falling through gas; and, finally, the data of Grace *et al.* (1978) and Sundell (1978) for bubbles rising through a stagnant liquid. The conditions of their experiments are described in table 1. It is evident from table 1 that the experimental data covers a broad range of liquid–liquid, liquid–gas and gas–liquid systems.

Table 1. Summary of various experiments on maximum fluid particle size

Reference	Dispersed/continuous fluids	ρ^*	$N\mu_c$	d_e^*
Merrington & Richardson (1947)	Liquid/gas	884–1442	1.1×10^{-3}–2.7×10^{-3}	3.73–4.63
Ryan (1978)	Liquid/gas	953	1.6×10^{-3}–3.5×10^{-3}	3.25–3.42
Finlay (1957)	Liquid/gas	767–2834	1.2×10^{-3}–2.6×10^{-3}	2.67–4.32
Krishna *et al.* (1959)	Liquid/liquid	1.15–3.32	1.4×10^{-3}–6.7×10^{-3}	3.74–5.65
Hu & Kintner (1955)	Liquid/liquid	1.64–3.33	2.4×10^{-3}–4.0×10^{-3}	3.50–4.38
Grace *et al.* (1976)	Liquid/liquid	0.75–1.36	3.8×10^{-2}–6.9	4.06–37.06
Grace *et al.* (1978)	Gas/liquid	10^{-3}–1.6×10^{-3}	4.5×10^{-2}–2.6	19.27–36.95
Sundell (1978)	Gas/liquid	1.41×10^{-3}	2.3×10^{-3}	30.98

The average deviation between the predicted and experimental values of $(d_e)_{max}$ varies from about +5.80% for Ryan's data to +29.55% for the Grace *et al.* data, with an overall mean deviation of $\pm 13.94\%$. Four of the systems studied by Hu & Kintner are similar to systems investigated by Krishna *et al.*, while two of the Finlay systems are essentially identical to the Merrington & Richardson systems. However, the mean deviation changes drastically between the Hu & Kintner and Krishna *et al.* data and between the Merrington & Richardson and Finlay data. Although there are some differences in the reported values of the fluid properties, a significant part of the discrepancy between predictions and theory arises from experimental scatter or bias.

Relatively high disagreements between the predicted and the Grace *et al.* data for liquid–liquid systems can be attributed to the viscosity effects of the continuous fluid. Although a viscosity correction has been made in the final correlation it is not expected that the correlation would be good for very viscous fluids. The experiments of Grace *et al.* cover a dynamic viscosity range of 0.0124–3.08 Ns/m. If this set of data, ∇ in figure 6, is excluded from the comparison the overall mean deviation decreases drastically.

Taking the experimental scatter and the very viscous fluids used for some experiments into consideration and, furthermore, recalling the approximate nature of the theory developed here, the agreement between the theoretical predictions and the experimental results is favorably good. The overall mean deviation between the predicted and experimental values of $(d_e)_{max}$ is $\pm 13.94\%$. The agreement with the experimental results indicates that the principal physical mechanisms involved are properly accounted for.

PRACTICAL BREAKUP CORRELATIONS

It is to be noted that the general breakup correlation developed above is not closed in the sense that it requires an iteration process to obtain the maximum fluid particle size. This may hinder its practical use. For practical applications the general criterion may be simplified for falling drops in gases, drops in a high relative velocity field, rising bubbles and falling or rising drops in liquids.

Freely falling drops in gaseous media

As discussed above $\theta_\omega = \pi/2$ for drops, and $C_e = 1$. Noting that $\rho^* \gg 1$ and $N\mu_c \ll 1$ for this case, the breakup criterion can be simplified for practical purposes. It may be shown that the criterion becomes

$$d_e^* + 0.26\mathrm{We} - 16 = 0. \qquad [33]$$

With the terminal velocity given by [19], We can be expressed in terms of d_e^*. Thus,

$$\mathrm{We} = 4d_e^*. \qquad [34]$$

In view of [34], [33] yields a very simple expression for d_e^*, as follows:

$$d_e^* \simeq 3.52. \qquad [35]$$

It is interesting to note that when the slight effects of ρ^* and $N\mu_c$ are neglected and newly found breakup criterion reduces to the classical We criterion as

$$\mathrm{We} = 4d_e^* = 14.08. \qquad [36]$$

In the case of a falling drop, the classical We criterion, as suggested by Hinze (1955), Sevik & Park (1973) and Haas (1964), is given by We $\simeq$ 22. This yields somewhat larger d_e^* values than those predicted from [35] or [36]. However, as can be observed from figure 6, a comparison of the present predictions with available experimental data indicate a very good agreement with the data of Merrington & Richardson (1947) and Ryan (1978) with a mean deviation of $\pm 8.2\%$. Somewhat poor predictions may be seen with Finlay's (1957) four data points. Including Finlay's data the mean deviation is ± 12.08.

Before closing this section it is interesting to compare the Grace *et al.* (1978) method, which was based on the Rayleigh–Taylor instability analysis, with the same experimental data set used here. Based on the numerical predictions presented by Grace *et al.* (1978), the mean deviation from the experimental data of Merrington & Richardson (1947), Ryan (1978) and Finlay (1957) is in the order of $\pm 31.85\%$. Comparison of this value with the present model mean deviation of $\pm 12.08\%$, using the same data set, indicates the effects of relative velocity on the interfacial stability. Although the breakup models used here and by Grace *et al.* (1978) are similar in nature, inclusion of the relative velocity effects considerably improves the predictions.

Drops in a high-velocity gas field

In this case, again, $\theta_\omega = \pi/2$, $\rho^* \gg 1$ and $N\mu_c \ll 1$; however, u_c is not uniquely expressed in terms of d_e. In order to de-couple We and d_e^*, a modified We, We_m, is defined as

$$\mathrm{We}_m \equiv \frac{\rho_c u_c^2}{(\sigma g\,|\Delta\rho\,|)^{1/2}} = \frac{\mathrm{We}}{d_e^*}. \qquad [37]$$

It may be shown from [33] that the criterion becomes

$$d_e^{*2} + 0.26\mathrm{We}_m d_e^* - 4 = 0. \qquad [38]$$

The breakup diameter for droplets in a high-velocity gas stream, as predicted by [38], is illustrated in figure 7. The predictions are also compared in the figure with those determined by the classical We criterion, We $\simeq$ 12–17, and by the Kataoka *et al.* (1983) correlation. The latter, which was obtained in collaboration with a large amount of experimental air–water data from Wicks & Dukler (1966), Cousins & Hewitt (1968) and Lindsted *et al.* (1978), is given in its original form as

$$\mathrm{We} = 0.031\mathrm{Re}_{cd}^{2/3}\left(\frac{\rho_c}{\rho_d}\right)^{-1/3}\left(\frac{\mu_c}{\mu_d}\right)^{2/3} \qquad [39]$$

where Re_{cd} is the continuous phase Reynolds number based on the hydraulic equivalent diameter of flow passage. For convenience Kataoka *et al.*'s correlation may be recast into the following form:

$$d_e^* = 0.031\left(\frac{d_h^*}{N\mu_d\,\mathrm{We}_m}\right)^{2/3} \qquad [40]$$

where d_h^* and $N\mu_d$, respectively, are the dimensionless hydraulic equivalent diameter of flow passage, $d_h^* \equiv d_h\,(g\,|\Delta\rho\,|/\sigma)^{1/2}$, and the dispersed phase viscosity number, $N\mu_d \equiv \mu_d/(\rho_c^2\sigma^3/g\,|\Delta\rho\,|)^{1/4}$. For the purpose of comparison, air–water properties at atmospheric conditions are used for evaluating $N\mu_d$ and the hydraulic diameter is treated as a parameter in figure 7.

It is important to note from figure 7 that as the hydraulic diameter increases predictions based on the data of Kataoka *et al.* (1983) approach the present model predictions in the general trend as well as numerical values. The agreement is remarkably good in relatively large flow channels. This is an expected result since the present model is based on the stability of a single particle in an infinite medium, neglecting the effects of surrounding particles and wall effects. On the other hand, the classical We criterion with the constant being 12 and 16, as shown in the figure, consistently underestimates the breakup diameter. Although general trends are similar to the present model, predictions of the We criterion are quite different from those obtained from the present model and the Kataoka *et al.* correlation. The classical We criterion and its predictions are discussed in more detail in Kataoka *et al.* (1983).

In conclusion, the comparison with the experimental data based correlation of Kataoka *et al.* (1983) indicates that, indeed, the proposed breakup mechanism has been the dominant factor in

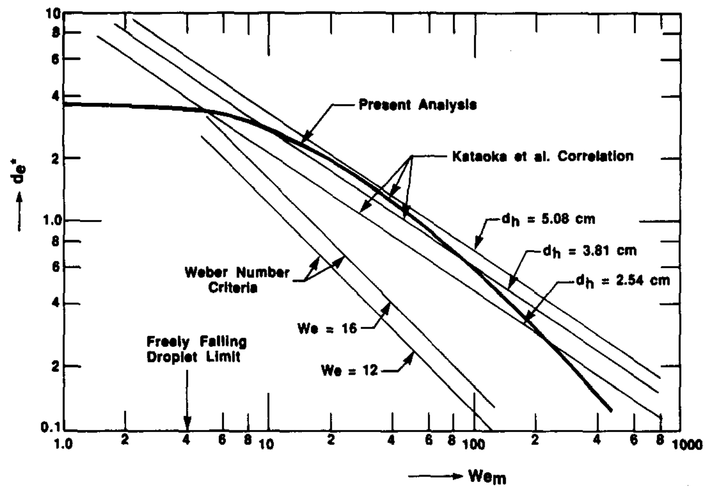

Figure 7. Breakup diameter correlation for drops in a high-velocity gas stream and comparison with other correlations.

determining the maximum fluid particle size. An investigation is presently underway to improve the present model to take into account the effects of surrounding particles and wall effects.

Rising bubbles in stagnant liquids

In view of [18], We can be expressed as

$$\mathrm{We} = \left(\frac{2C_e}{9}\right) d_e^{*2}. \qquad [41]$$

Noting that $\rho^* \ll 1$ for bubbles, and using [41] in [32], the breakup criterion for bubbles becomes

$$d_e^{*2} = \frac{\left[4\left(\frac{2\pi}{\theta_\omega}\right)^4\left(\frac{1}{C_e}\right)^2\right]}{\left[0.25\left(\frac{2\pi}{\theta_\omega}\right)^2 \sin^2\left(\frac{\theta_\omega}{2}\right)\rho^* + 0.5\left(\frac{2\pi}{\theta_\omega}\right) - [0.584\,(1+\mathrm{N}\mu_c)^{0.36}]\left\{\ln\left[\frac{\left|\tan\left(\frac{\theta_\omega}{2}\right)\right|}{\left|\tan\left(\frac{\theta_\omega}{4}\right)\right|}\right]\right\}\right]}, \qquad [42]$$

where θ_ω is given as a function of Re_c by [16].

Basically, it seems there are three independent variables, namely θ_ω, ρ^* and $\mathrm{N}\mu_c$, affecting d_e^*. However, with the terminal velocity for large bubbles given by [18], Re_c can be expressed as

$$\mathrm{Re}_c = \left(\frac{2}{3}\right)\left(\frac{C_e}{2}\right)^{1/2}\left(\frac{d_e^*}{\mathrm{N}\mu_c}\right)^{3/2} \qquad [43]$$

This indicates that there are two independent variables rather than three. Furthermore, noting that the term containing ρ^* is much smaller than the other two terms in [42], and in view of [41], it may be observed that $d_e^* = f(\mathrm{N}\mu_c)$ only. Hence an even simpler correlation can be developed in this functional form. It may be shown that

$$d_e^* = 27.07(1 + \mathrm{N}\mu_c)^{0.83} \qquad [44]$$

is a very good representation of [42] for the bubble breakup diameter.

Equation [44] is very simple to use when compared to [42]. It is presented in figure 8 along with the Grace *et al.* (1978) data. The breakup diameter slightly increases as the continuous phase viscosity number increases. As demonstrated by Kocamustafaogullari (1985), this may be attributed to the smoothing effect of the interfacial shear on the growth of interfacial waves.

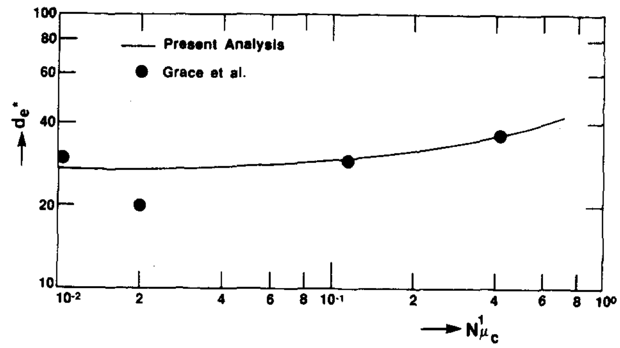

Figure 8. Breakup diameter correlation for bubbles and comparison with experimental observations.

Unfortunately, the amount of experimental data available in the literature is very limited for this case—making it difficult to check the validity of [44] in more detail.

Falling or rising drops in stagnant liquids

Recalling that $\theta_\omega = \pi/2$ for drops, the breakup criterion can be simplified to obtain

$$d_e^{*2} + \left[4.5\left(\frac{\rho^*}{1+\rho^*}\right) - 0.35\left(\frac{2+3\rho^*}{1+\rho^*}\right)^{2.27}(1+\mathrm{N}\mu_c)^{0.36}\right]\mathrm{We} - 16 = 0. \qquad [45]$$

With the terminal velocity given by [20] and Ar defined by [21], We can be expressed as

$$\mathrm{We} = 0.25\,\frac{\mathrm{N}\mu_c^2}{d_e^*}\left\{\left[F^2 + 2\left(\frac{d_e^{*3}}{\mathrm{N}\mu_c^2}\right)\right]^{1/2} - F\right\}^2. \qquad [46]$$

It is important to note that as (μ_d/μ_c) increases from 0 to ∞, F varies only from 6 to 9, indicating that the effect of the (μ_d/μ_c) ratio on We is insignificant. Therefore, We can be approximated from [46] as follows:

$$\mathrm{We} \simeq 0.5 d_e^{*2}. \qquad [47]$$

In view of [47] the breakup criterion for drops in liquids can be further simplified resulting in the following expression for d_e^*:

$$d_e^{*2} = \frac{16}{1 + 0.5\left[4.5\left(\frac{\rho^*}{1+\rho^*}\right) - 0.35\left(\frac{2+3\rho^*}{1+\rho^*}\right)^{2.27}(1+\mathrm{N}\mu_c)^{0.36}\right]}. \qquad [48]$$

This equation can be safely used for $\mathrm{N}\mu_c \leqslant 4$ for predicting the breakup diameter of freely falling or rising liquid droplets in stagnant liquids. The influence of $\mathrm{N}\mu_c$ is illustrated in figure 9. The

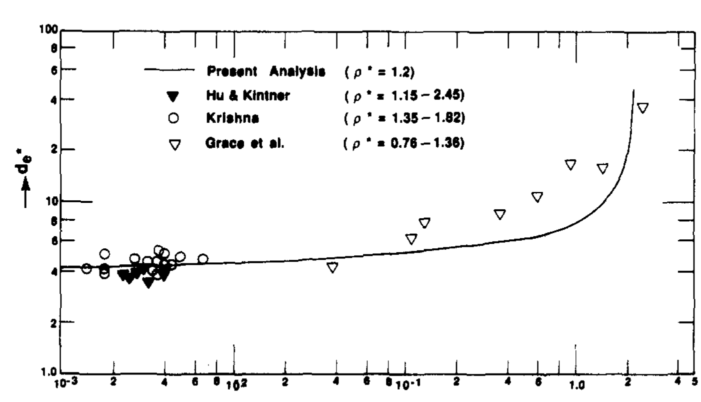

Figure 9. Breakup diameter correlation for drops in liquids and comparison with experimental observations.

dimensionless breakup diameter slightly increases as the continuous phase viscosity number increases. This behavior is similar to the bubble breakup behavior, as illustrated in figure 8. Comparisons between predicted and experimental observations are good for low values of $N\mu_c$, which is the case for most practical applications. However, when $N\mu_c \geqslant 4$, [45] is recommended with We expressed by [46].

SUMMARY AND CONCLUSIONS

Based on the combination of Kelvin–Helmholtz and Rayleigh–Taylor stability theory, a simple mechanistic model is developed to describe the breakup of drops and bubbles falling or rising freely in a stagnant media. Breakup is predicted to occur if the growth of disturbances on the leading front is rapid enough relative to the rate at which the disturbance is propagated around the interface. In collaboration with a large number of experimental data for liquid–gas, liquid–liquid and gas–liquid systems, a general correlation is developed to predict the maximum stable particle size in a stagnant fluid. The results are extended to predict the maximum droplet size in a high-velocity gas field. Important dimensionless parameters affecting the breakup process are properly identified. They are Weber number, continuous phase viscosity number, density and viscosity ratio groups. For each case, the importance of these groups is assessed.

Predicted values of the maximum particle size are compared with experimental data. An average deviation of $\pm 13.94\%$ between predicted and experimental values is observed. Considering the various simplifications made in the analysis the agreement is favorably good, and much better than that obtained from the Rayleigh–Taylor instability.

For practical applications, the general breakup criterion is simplified for four separate cases, i.e. for freely falling drops in a gas, droplets in a high-velocity gas stream, freely rising bubbles and, finally, for freely falling or rising drops in liquids. Simple correlations are developed for each case. With the proper form of the terminal velocity it is demonstrated that the dimensionless breakup diameter can be expressed in terms of ρ^*, $N\mu_c$ and (μ_d/μ_c). Depending on the bubbles or drops effective dimensionless groups can be further reduced as given by [35], [38], [44] and [48].

The theoretical model developed in this study is clearly approximate in nature, including such simplifications as the treatment of growth of three-dimensional disturbances on curved interfaces by means of two-dimensional, small-amplitude waves on a flat interface. Despite these approximations, the agreement with experimental results indicates that the principal physical mechanisms involved are properly accounted for.

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